$a^2 + b^2 = c^2$

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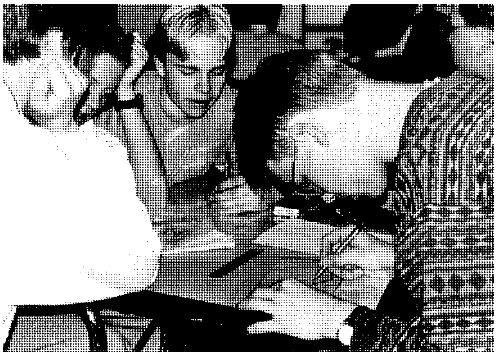


Pythagorean triangles
i.e. the sides are integers.

We have been working with pythagorean triangles. During the investigation of the subject we also dealed with Archemedean triangles. Furthermore we had a contest. All this you may read more about is the following.

$a^2 + b^2 = (b + 1)^2$

The purpose is to determine the length of the sides in a rectangular triangle, in which the hypotenuse is 1 bigger then the longest side.

$$a^2 + b^2 = (b + 1)^2$$
$$a^2 + b^2 = b^2 + 2b + 1$$
$$a^2 = 2b + 1$$
$$b = (a^2 - 1)/2$$

Then $c = b + 1 \rightarrow c = (a^2 + 1)/2$

We can add up these results :

$$(a, b, c) = \left(a, (a^2 - 1)/2, (a^2 + 1)/2\right),$$
$$a = 3, 5, 7, 9, \dots$$

Here are some examples :

$$3^2 + 4^2 = 5^2$$
$$5^2 + 12^2 = 13^2$$
$$7^2 + 24^2 = 25^2$$

a cannot be an even number. If you try to put in an even number into the formulae for b and c, the result will not be an integer.

$a^2 + b^2 = (b + 2)^2$

This formula is used to find integer sides on pythagorean triangles. You can find the sides of the triangles with a hypotenuse c that is 2 bigger than b.

From Pythagoras :

$$a^2 + b^2 = c^2$$
$$a^2 + b^2 = (b+2)^2$$
$$a^2 + b^2 = b^2 + 4b$$
$$a^2 + b^2 = b^2 + 4b + 4$$
$$b = (a^2 - 4)/4$$

Then

$$c = b + 2$$
$$c = (a^2 + 4)/4$$

We came to these results :

$$(a, b, c) = (a, (a^2 - 4)/4, (a^2 + 4)/4)$$

Examples :

$$4^2 + 3^2 = 5^2$$
$$8^2 + 15^2 = 17^2$$
$$12^2 + 35^2 = 37^2$$

a must be divisible by 4 (a = 4, 8, 12, 16, 20 …). If a is odd, b and c will not become integers. If a is divisible by 2 but not by 4 the triple (a, b, c) have a common factor.

For instance a = 6 gives the triple (6, 8, 10) ~ (3, 4, 5).

Our purpose is to find a systematic way to determine all primitive pythagorian triples

We want to prove that :

(a, b, c) can be written as

$$(2pq, p^2 - q^2, p^2 + q^2),$$

when the conditions for p and q are :
- not both odd
- $p > q$
- no common factors

We assume that a is even and b, c are odd.

PYTHAGORAS

$$a^2 = c^2 - b^2$$
$$a^2 = (c + b)(c - b)$$

divided by two^2 :

$$(a/2)^2 = ((c + b)/2).((c - b)/2)$$
$$(c + b)/2 = p^2$$

p and q have to be squares so that $(a/2)^2$ can be a square

$$(c - b)/2 = q^2$$
$$p^2 + q^2 = ((c + b)/2) + ((c - b)/2) = c$$
$$p^2 - q^2 = ((c + b)/2) - ((c - b)/2) = b$$
$$(a/2)^2 = a^2/4$$
$$a^2/4 = p^2.q^2$$
$$a^2 = 4\,(p^2 \,.\, q^2)$$

squareroot

$$a = 2\,(p \,.\, q)$$

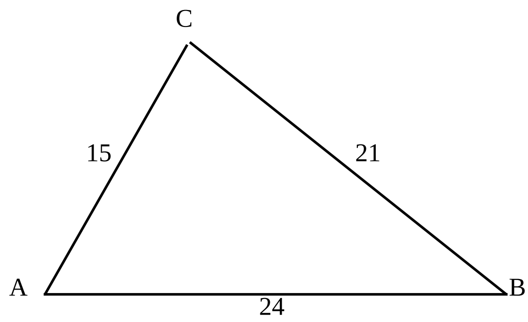
Archimedean triangles

A triangle with one angle π/3 (60°) and with integer sides is called an archimedean triangle. To find a general formula for these triangles, our startingpoint is the cosine-relations.

Cosine :

$$a^2 = b^2 + c^2 - 2bc.cos(60°)$$

where cos (60°) = 1/2



Formula :

$$a^2 = b^2 + c^2 - bc$$
$$4a^2 = 4b^2 + 4c^2 - 4bc$$
$$4a^2 - 4c^2 - b^2 + 4bc = 3b^2$$
$$(4a^2) - (4c^2 - 4bc + b^2) = 3b^2$$
$$(2a)^2 - (2c - b)^2 = 3b^2$$
$$(2a + 2c - b).(2a - 2c + b) = 3b^2$$
$$3p^2q^2 = 3b^2$$

$$a = (3p^2 + q^2)/4$$
$$b = p.q$$
$$c = (3p^2 - q^2 + 2pq)/4$$

We found some archimedean triangles from the formula and from this we can conclude that there is no system what so ever.

Examples of archimedean triangles :

$p = q = 1$	$(a, b, c) = (1, 1, 1)$
$p = 3, q = 1$	$(a, b, c) = (7, 3, 8)$
$p = 3, q = 5$	$(a, b, c) = (13, 15, 8)$
$p = 5, q = 1$	$(a, b, c) = (19, 5, 21)$
$p = 5, q = 3$	$(a, b, c) = (21, 15, 24)$

Contest :

We made a contest, where you were to find *c* and *b*, when *a* equals 1994. *c* and *b* were to be integers. The prize winner of a FG-sweat-shirt was Siek-Hor Lim.

$$(a/2)^2 = ((c + b)/2).((c - b)/2)$$
$$(1994/2)^2 = 997^2$$
$$(c + b)/2 = 997^2$$
$$(c - b)/2 = 1$$
$$\Rightarrow c - 1 = 997^2 = 994009$$

$c - b = 2$	$994009 + 1 = 994010$
$b + 2 = c$	$994009 - 1 = 994008$

(The difference is two)

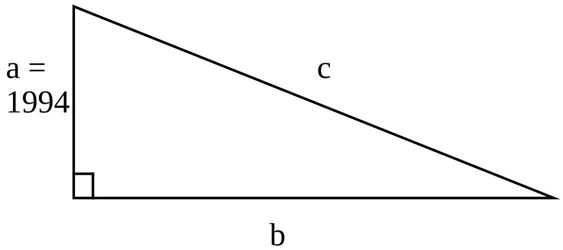
$$(a, b, c) = (1994, 994008, 994010)$$

affiché sur un des panneaux par les élèves danois pendant le congrès, au Palais de la découverte :

* CONTEST *

You are to find the sides b and c satisfying the Pythagorean theorem $a^2 + b^2 = c^2$ in which $a = 1994$

The numbers have to be integers.



If you have any solutions, please give them to one of us.

The winner will get a prize !

We will announce the winner + give the procedure and solution Monday about 2 pm.